Simulation of gas dynamics in heat engines of complex shapes

Vetchanin E.V.*, Tenenev V.A.

Izhevsk State Technical University, Izhevsk, Russia

Abstract

An algorithm for designing a block computational grid by the complex method of boundary elements is given. The results of numerical simulation of the flow in a region with complex geometry using an orthogonal grid and a multigrid method are presented.

Keywords: algorithm; boundary element; gas dynamics; numerical modeling

1. Introduction

For a solid propellant rocket engine, when assessing the energy-mass characteristics, the values of gas-dynamic parameters in the combustion chamber and nozzle are important. Even for axisymmetric structures, the flow region has a complex geometry. Complex geometry results from the presence of a nozzle pushed into the combustion chamber ("recessed" nozzle), existing radial grooves, and open ends of the charges. The calculation of flows in such areas is associated with computational difficulties due to the complex shape of the boundaries. In this paper, we consider an approach to the calculation of axisymmetric flows based on the use of curvilinear orthogonal grids and the multigrid method

2. Materials and methods

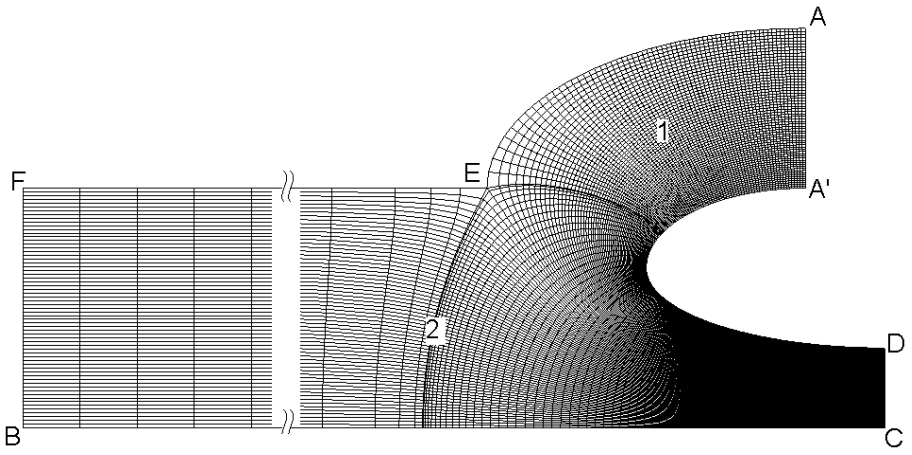

Figure 1. Physical area covered by the block mesh

For the complex geometry of the flow area, a block partition into conjugate subdomains is proposed. The general view of the area is shown in Figure 1. The area under consideration has a complex geometry and it is initially necessary to build a grid. A computational grid close to orthogonal

*Corresponding author
email: eugene186@mail.ru
doi: https://doi.org/10.23877/MS.TS.39.004
Published online: 15 June 2017

© Vetchanin E.V., Tenenev V.A.

can be constructed using the complex boundary element method (CBEM). The mathematical foundations of the method are described in detail in [1], in [2] its advantages are described in relation to other meshing methods.

CBEM builds a conformal mapping of the original physical area Ω with border *B* into the computational domain *D* with border *G*, grid coordinate lines will be streamlines and isopotential lines of some potential flow.

When using CBEM, the boundary of the region is approximated by a broken line with the coordinates of the nodes $z_k = x_k + iy_k$, $k = 1, L$ in the complex area *Z*.

On the boundary sections, the boundary values of the functions φ and ψ

$$\begin{aligned} &AB: \varphi = 0 \\ &BC: \psi = 0 \\ &DA: \psi = 1 \\ &CD: \psi = \frac{s - s_C}{s_D - s_C}, \end{aligned} \qquad (1)$$

where s_C - distance measured along the border of the area Ω from the point *C*.

φ and ψ and are the real and imaginary parts of some analytic Ω functions $\omega = \varphi + i\psi$, for which the Cauchy integral:

$$\omega(z_0) = \frac{1}{2\pi i}\int_{\Gamma}\frac{\omega(\rho)d\rho}{\rho - z_0} \qquad (2)$$

Using Linear Approximation integral (2), as shown in [1], is written as

$$\omega(z_0) = \frac{1}{2\pi i}\sum_{k=1}^{L}\frac{(z_0 - z_k)\omega_{k+1} - (z_0 - z_{k+1})\omega_k}{z_{k+1} - z_k} \times ln\left(\frac{z_{k+1} - z_0}{z_k - z_0}\right) \qquad (3)$$

For known values ω_k expression (3) defines a function that is analytical in the domain.

Conditions (1) define at each boundary node one of the functions φ or ψ. Therefore, it is necessary to determine the unknown values of these functions. For this point z_0 rushes to each boundary node z_k according to (3). Because of passing to the limit, we get *L* linear algebraic equations with respect to *L* unknown boundary values of functions φ and ψ.

After determining the boundary values, lines of equal levels are drawn φ or ψ, that cover the computational area *D*. To define mesh nodes in the physical area Ω it is necessary to solve the equations

$$x_{\varphi\varphi} + x_{\psi\psi} = 0 \qquad (4)$$

$$y_{\varphi\varphi} + y_{\psi\psi} = 0 \qquad (5)$$

Then for an analytic domain *D* ($\omega \in D$) functuions $z(\varphi,\psi) = x(\varphi,\psi) + iy(\varphi,\psi)$ the Cauchy integral formula is also valid

$$z(w_0) = \frac{1}{2\pi i}\int_{C}\frac{z(\rho)d\rho}{\rho - \omega_0} \qquad (6)$$

Values $\omega_0 = \varphi_m + i\psi_n$ are defined using predetermined feature levels φ_m; ψ_n; $m = 1, M$; $n = 1, N$. Then the coordinates of the internal nodes of the curvilinear orthogonal grid $z_0 = x_{mn} + iy_{mn}$ are defined by the expression

$$z(\omega_0)=\frac{1}{2\pi i}\sum_{k=1}^{L}\frac{(\omega_0-\omega_k)z_{k+1}-(\omega_0-\omega_{k+1})z_k}{\omega_{k+1}-\omega_k}\ln\left(\frac{\omega_{k+1}-\omega_0}{\omega_k-\omega_0}\right) \quad (7)$$

When calculating unknown boundary values ω_k their values do not necessarily coincide with the given ones, which is associated with approximation and calculation errors. Because you want to map the physical area Ω into a rectangle D, then the boundary values found are corrected accordingly. The coordinate lines obtained by formula (7) may overlap near strongly curved boundaries, so they should be considered as an initial approximation. The coordinates of internal grid nodes are refined by numerically solving Eqs. (4) and (5) using a strongly implicit procedure (SIP) [3].

This approach is effective for not very complex areas. For the area under consideration, using the described approach in the original form, it is not possible to construct a high-quality grid, some of the coordinate lines will be very sparse. In this regard, it is proposed to build a block grid. The area is conditionally divided into several blocks Figure 1 by some boundary, which will be determined during the grid construction

When using CBEM, the original physical domain will be displayed in the computational domain, which is also a block domain, but each block is a rectangle. The view of this area is shown in Figure 2.

To construct a block grid, it is necessary to set the following boundary values of the functions φ and ψ:

$$\begin{aligned}
&CD:\varphi=\varphi_0\\
&DA':\psi=2\\
&A'A:\psi=\frac{s-s_A}{s_{A'}-s_A}+1\\
&AE:\psi=1\\
&EF:\psi=1\\
&FB:\psi=\frac{s-s_B}{s_F-s_B}\\
&BC:\psi=0.
\end{aligned} \quad (8)$$

The design of the grid is complicated by the need to join blocks. Figure 2 below shows the details.

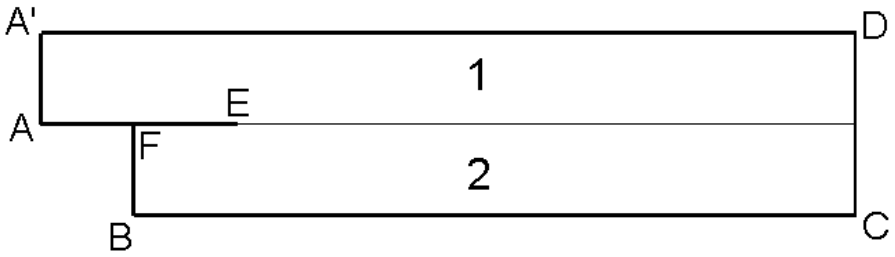

Figure 2. Calculation area (thick lines – boundary, thin line – block border)

3. Results and discussion

The control volume method [4] was chosen to calculate the gas dynamics. In this method, the most costly, from the point of view of computer time, is the calculation of corrections to pressure, since the system of linear algebraic equations (SLAE) for the corrections is ill-conditioned and has a rather large dimension. The conditionality of the system worsens when the grid is denser and the presence of strongly changing physical parameters. In computational gas dynamics, these two factors act, as a rule, jointly. In this paper, we consider the flow in the combustion chamber and near-nozzle section, where $M \le 0{,}3$, the problem statement is given in [5].

Table 1. Comparative data for different approaches for solving the Poisson problem

Grid size	Conjugate gradient method with pre-pinning, s	Multigrid method, s	Full multigrid method, s
512×512	15.8236	1.38068	1.06193
1024×1024	105.923	5.9333	3.887
2048×2048	842.913	22.9758	13.7384

The successful application of the conjugate gradient method with preconditioning for the numerical solution of ill-conditioned high-dimensional SLAEs is known in [5], [6]. Several papers [7], [8], [9] describe multigrid methods that were used in solving problems of gas dynamics in [10].

Table 1 and Figure 3 show the costs of difference methods when solving the Poisson problem in the unit square on grids of different sizes. The stopping criterion was the condition $\max_{ij}|r_{ij}| < 10^{-6}$, where r_{ij} - knot residual ij. Testing was carried out on a computer with the following characteristics: CPU AMD ATHLON 64 X2 5600+, RAM Samsung DDR-II 2GB (PC2-6400) 800MHz.

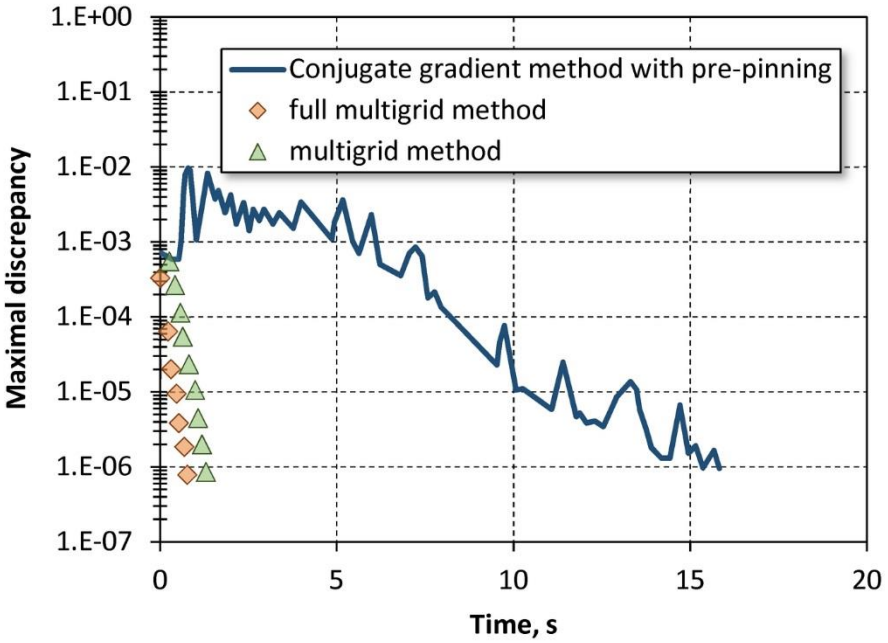

Figure 3. Behavior of Methods on a Uniform Grid 512×512

Multigrid methods quickly reach the given accuracy of the solution, and the full multigrid method converges faster due to the search for an initial approximation. The conjugate gradient method with preconditioning reaches convergence in 16 s, which is almost 15 times longer than the full multigrid method.

For calculations, we used a block curvilinear orthogonal grid constructed by complex boundary element method (CBEM). The size of each block was 128×64 cells. As a result of calculations, the velocity field in the near-nozzle part was obtained, shown in Figure 4 and Figure 5.

For the considered problem, when using the full multigrid method, convergence is achieved on average in 76 s, of which 47 s are spent on average for calculating pressure corrections. When using the conjugate gradient method with preconditioning, it takes 168 s, of which an average of 147 s is spent on calculating pressure corrections.

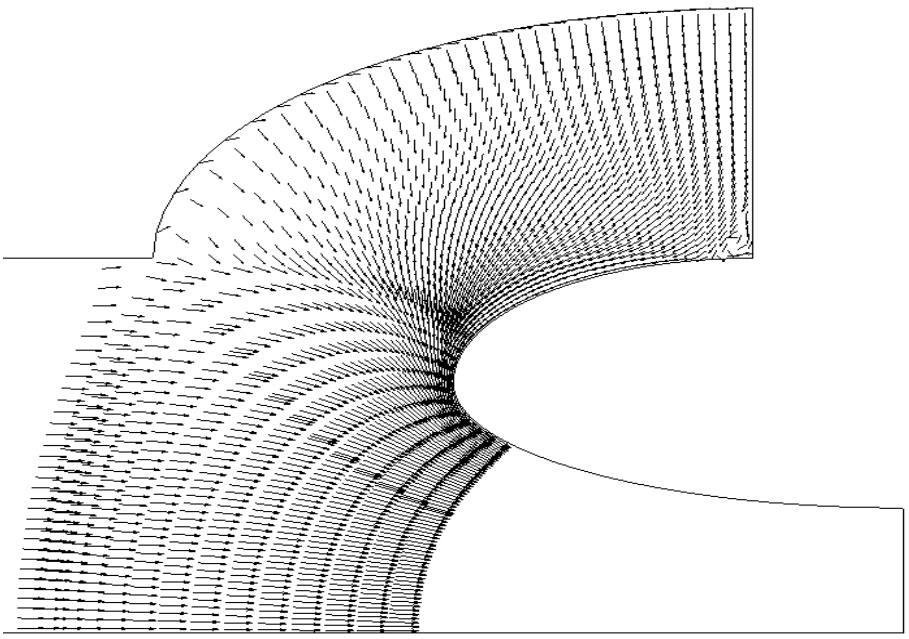

Figure 4. Flow in near-nozzle part

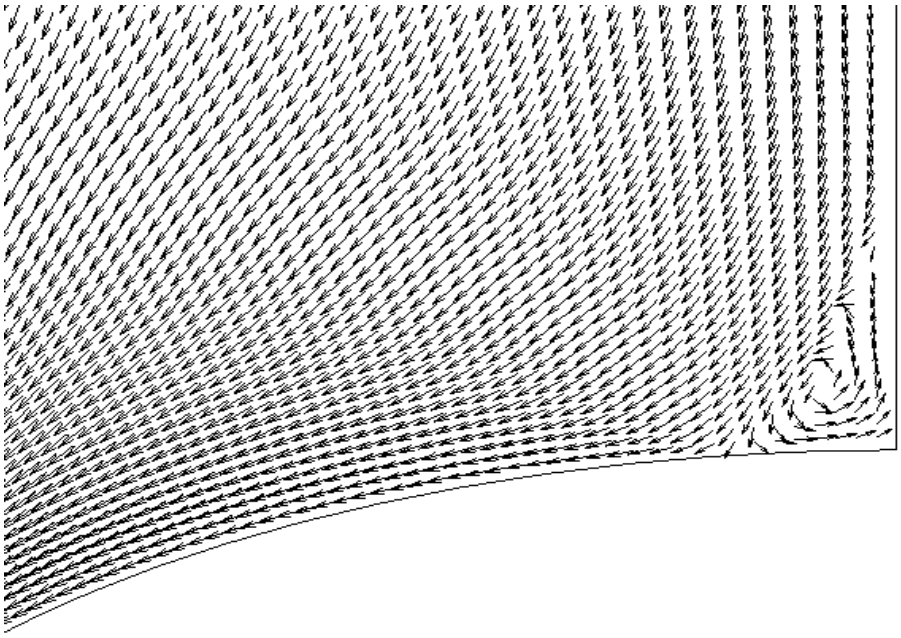

Figure 5. Vortex zone

4. Conclusions

This paper shows the possibility of constructing block grids using the complex boundary element method. The grid constructed by this method will be close to orthogonal, since its coordinate lines are streamlines and isopotential lines of some potential flow.

The use of multigrid methods significantly speeds up the solution of gas dynamics problems. In addition, they are more resistant to strongly changing coefficients. In this work, a full multigrid method was used, the running time of which linearly depends on the dimension of the system. For finer meshes and the 3D case, multi-grid methods will yield an even greater reduction in the required computer time.

© Vetchanin E.V., Tenenev V.A.

References

[1]. Gromadka P.T., Lei Ch. Complex method of boundary elements in applied sciences. – M.: Mir, 1990. 303 p.

[2]. Tenenev V.A., Rusyak I.G. Numerical solution of problems of hydrodynamics and heat transfer in areas of complex shape: Educational and methodological manual. - Izhevsk: IzhGTU Publishing House, 1996. - 60 p.

[3]. E.V. Vetchanin Comparative efficiency of multigrid methods for solving systems of difference equations // Intelligent systems in production.

[4]. Patankar S. Numerical methods for solving problems of heat transfer and fluid dynamics. – M.: Energoizdat, 1984. – 150 p.

[5]. Bendersky B.Ya., Tenenev V.A. Experimental-numerical study of flows in axisymmetric channels of complex shape with injection. Izv. RAS MJG No. 2, 2001.

[6]. Bendersky B.Ya., Tenenev V.A. Spatial subsonic flows in regions with complex geometry. //Mathematical modeling, v.13, №8, 2001.p.47-52.

[7]. Fedorenko R.P. Introduction to Computational Physics: Proc. allowance: For universities. - M .: Publishing House of Moscow. physics and technology in-ta, 1994. - 528 p.

[8]. P. Wesseling An introduction to multigrid methods

[9]. U. Trottenberg, C. W. Oosterlee, A. Schüller Multigrid

[10]. Neither R.-H. Multigrid method for solving the Euler equations //Aero-kosm. Technique. - 1983. N6. pp.61-69